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*Tm Tool v1.0b*

*DNA.UTAH.EDU*

# ***Software Methods and Equations***

**V1.3**



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HEALTH SCIENCES

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## METHODS

### 0.1 $T_M$

$T_m$  is the melting temperature of a DNA duplex in a thermodynamically closed system. This allows the use of thermodynamic parameters to define it. We start with two different definitions of Gibbs free energy.

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta G = -RT\ln K$$

### 0.2 SUBSTITUTION OF $\Delta G$

$$-RT\ln K = \Delta H - T\Delta S$$

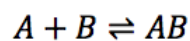
$$\therefore T = \frac{\Delta H}{\Delta S - R\ln K}$$

### 0.3 $\Delta H$ AND $\Delta S$

The  $\Delta H$  and  $\Delta S$  terms are sums of nearest neighbor thermodynamic parameters. The most common set is Santalucia-Hicks (2004), but many others exist.

### 0.4 $K$

$K$  is the equilibrium constant. For DNA there are two strands that are in the single strand (A, B) or double strand (AB) state.



$$K = \frac{[AB]}{[A][B]}$$

Substitute  $K$

$$T = \frac{\Delta H}{\Delta S - R\ln\left(\frac{[AB]}{[A][B]}\right)} = \frac{\Delta H}{\Delta S + R\ln\left(\frac{[A][B]}{[AB]}\right)}$$

### 0.5 CONCENTRATION FOR DIFFERENT CALCULATION TYPES

#1  $T_m$  is defined as 50% conversion to the double strand state, where the '50' subscript indicates 50% of the maximum possible concentration.

What is  $\left(\frac{[A_{50}][B_{50}]}{[AB_{50}]}\right)$  ?

If  $[A] \geq [B]$ , then the maximum concentration of the duplex that can be formed is  $[B]/2$ .

$$\left(\frac{[A_{50}][B_{50}]}{[AB_{50}]}\right) = \frac{\left([A] - \frac{[B]}{2}\right) \left(\frac{C[B]}{2}\right)}{\left(\frac{C_B}{2}\right)} = [A] - \frac{[B]}{2}$$

When  $[A]=[B]$ :

$$\left(\frac{[A_{50}][B_{50}]}{[AB_{50}]}\right) = \frac{[A]}{2}$$

This is equivalent to  $([A] + [B]) / 2$ , sometimes written as  $(Ct / 4)$  in other publications.

#2 For Primer  $T_m$  at the beginning of PCR (no probe):

$$\begin{aligned} [A] - \frac{[B]}{2} &= [primer] - \frac{[template]}{2} \\ [primer] &\gg [template] \\ &= [primer] \end{aligned}$$

#3 For Probe  $T_m$  at the end of PCR, we need to first estimate the concentration of the available product strand complementary to the probe. This is difficult, because it varies according to the experiment performed. As a default we use 60% of the excess primer remaining after duplex product formation. That is:

$$\text{Available product} = 0.6([excess primer] - [limiting primer])$$

If  $0.6([excess primer] - [limiting primer]) > [probe]$ :

$$[A] - \frac{[B]}{2} = 0.6([excess primer] - [limiting primer]) - \frac{[probe]}{2}$$

If  $[probe] > (0.6([excess primer] - [limiting primer]))$ :

$$[A] - \frac{[B]}{2} = [probe] - \frac{0.6([excess primer] - [limiting primer])}{2}$$

## 0.6 SALT CORRECTION

- #1  $\Delta S$  is adjusted with respect to salt in the buffer solution when predicting  $T_m$ . Monovalent cations  $[Mono^+]$  and magnesium cations  $[Mg^{++}]$  will affect  $\Delta S$  and thus modify  $T_m$ . The common salt correction published in Santalucia 2004 is as follows ( $N$  is the number of bases of the sequence):

$$(0.368)(N - 1) \ln \left( [mono^+] + 3.795\sqrt{[free Mg^{++}]} \right)$$

- #2 Owczarzy et. al. (2008) have published their own salt correction that is dependent on

$$\frac{\sqrt{Mg^{++}}}{Mono^+} \text{ with 3 possible cases: A, B, \& C.}$$

**A)**  $0.22 < \frac{\sqrt{Mg^{++}}}{Mono^+} < 6.0$

The following equation is used:

$$\frac{1}{T_m(Mg^{2+})} = \frac{1}{T_m(1 M Na^+)} + a + b \ln[Mg^{2+}] + f_{GC}(c + d \ln[Mg^{2+}]) + \frac{1}{2(N_{bp} - 1)}[e + f \ln[Mg^{2+}] + g(\ln[Mg^{2+}])^2]$$

Coefficients are used for  $b, c, e, f$ :

$b$	$-9.11 \times 10^{-6}$
$c$	$6.26 \times 10^{-5}$
$e$	$-4.82 \times 10^{-4}$
$f$	$5.25 \times 10^{-4}$

Where the parameters  $a, d, g$  are defined as:

$$a = 3.92 \times 10^{-5} (0.843 - 0.352\sqrt{[Mon^+]} \times \ln[Mon^+])$$

$$d = 1.42 \times 10^{-5} [1.279 - 4.03 \times 10^{-3} \ln[Mon^+] - 8.03 \times 10^{-3} (\ln[Mon^+])^2]$$

$$g = 8.31 \times 10^{-5} [0.486 - 0.258 \ln[Mon^+] + 5.25 \times 10^{-3} (\ln[Mon^+])^3]$$

**B)**  $\frac{\sqrt{Mg^{++}}}{Mono^+} > 6.0$  or  $[Mono^+] = 0.0$

$$\frac{1}{T_m(Mg^{2+})} = \frac{1}{T_m(1\text{ M Na}^+)} + a + b \ln[Mg^{2+}] + f_{GC}(c + d \ln[Mg^{2+}]) + \frac{1}{2(N_{bp} - 1)}[e + f \ln[Mg^{2+}] + g(\ln[Mg^{2+}])^2]$$

With parameters as follows:

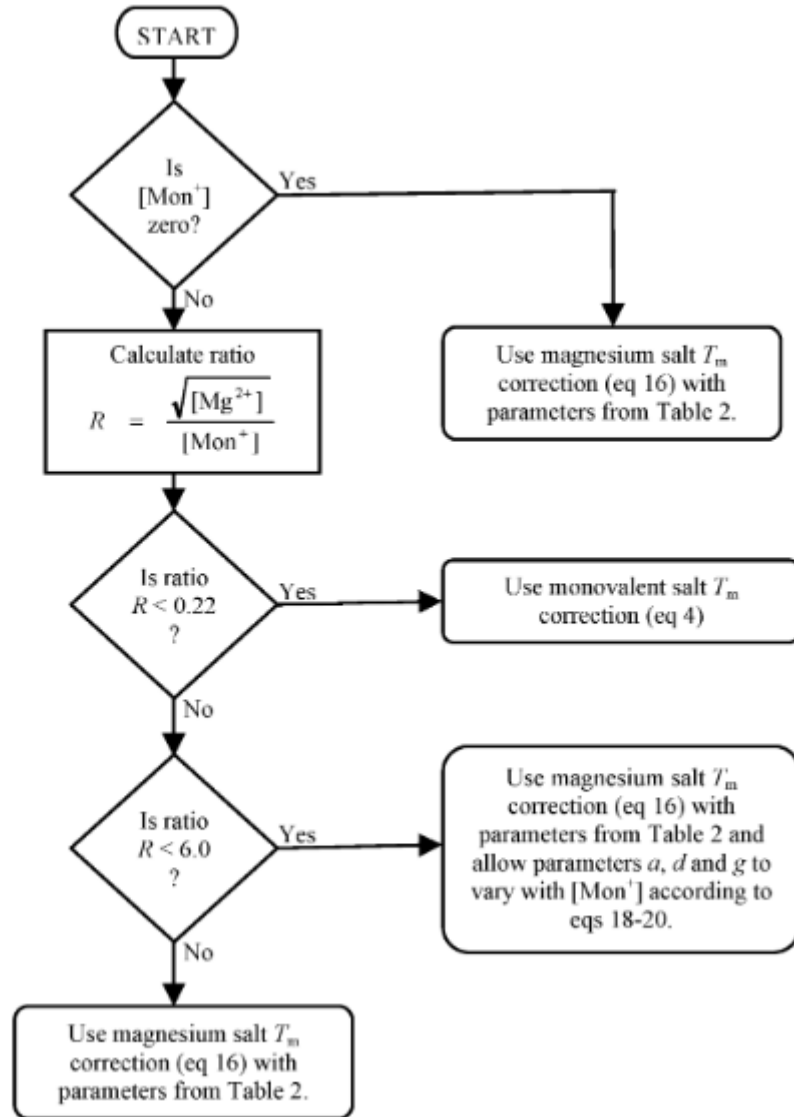
<i>a</i>	$3.92 \times 10^{-5}$
<i>b</i>	$-9.11 \times 10^{-6}$
<i>c</i>	$6.26 \times 10^{-5}$
<i>d</i>	$1.42 \times 10^{-5}$
<i>e</i>	$-4.82 \times 10^{-4}$
<i>f</i>	$5.25 \times 10^{-4}$
<i>g</i>	$8.31 \times 10^{-5}$

**C)**  $\frac{\sqrt{Mg^{++}}}{Mono^+} < 0.22$

The following is equation used:

$$\frac{1}{T_m(Mon^+)} = \frac{1}{T_m(1\text{ M Na}^+)} + (4.29f_{GC} - 3.95) \times 10^{-5} \ln[Mon^+] + 9.40 \times 10^{-6} (\ln[Mon^+])^2$$

The published work (cited in references) also includes a useful flowchart to easily illustrated the 3 cases and which equation is most applicable to the reader:



## 1 FULL T<sub>M</sub> EQUATION

Units:

ΔH in kcal/mole

ΔS in cal / (K \* mole)

R (gas constant) as 1.9872 cal/K•mol

Salt Correction as cal/mol

-273.15 as K to °C conversion.

[A] and [B] in mol with [A] greater or equal to [B]

T<sub>m</sub> in °C

$$T_m = \frac{\Delta H \cdot 1000}{\Delta S + (\text{salt correction}) + 1.9872 \ln \left( [A] - \frac{[B]}{2} \right)} - 273.15$$



## 2 REFERENCES

- Owczarzy, R., Moreira, B. G., You, Y., Behlke, M. A., & Walder, J. A. (2008). Predicting stability of DNA duplexes in solutions containing magnesium and monovalent cations. *Biochemistry*, *47*(19), 5336-5353. doi: 10.1021/bi702363u
- SantaLucia, J., Jr. (1998). A unified view of polymer, dumbbell, and oligonucleotide DNA nearest-neighbor thermodynamics. *Proc Natl Acad Sci U S A*, *95*(4), 1460-1465.
- SantaLucia, J., Jr., & Hicks, D. (2004). The thermodynamics of DNA structural motifs. *Annu Rev Biophys Biomol Struct*, *33*, 415-440. doi: 10.1146/annurev.biophys.32.110601.141800
- Sugimoto, N., Nakano, S., Yoneyama, M., & Honda, K. (1996). Improved thermodynamic parameters and helix initiation factor to predict stability of DNA duplexes. *Nucleic Acids Res*, *24*(22), 4501-4505.
- Weber, G. (2015). Optimization method for obtaining nearest-neighbour DNA entropies and enthalpies directly from melting temperatures. *Bioinformatics*, *31*(6), 871-877. doi: 10.1093/bioinformatics/btu751

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